

# **University Calculus: A New Perspective**

**For Any University and College Program**

**Volume 1**

**Spring Seeds**

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# Preface

The first chapter of this book gives preparatory knowledge on which calculus depends. The second chapter explains the concept of limit and its definition from a dynamic point of view. With this basic knowledge, further study of calculus becomes concrete and straightforward.

We have written this calculus textbook to use in our teaching, and it has received high praise from students. Students found the concepts and theories in this book to be very clear and logical, and thus, learning calculus became very easy, which is easier than learning other mathematics courses. Students also achieved excellent grades in the study of calculus. The teaching effect is remarkable. It can be said that this book has completely changed the difficult situation of teaching calculus and made learning calculus easy.

Calculus is one of the outstanding achievements of the human intellect. It is the foundation of modern mathematics. All advanced mathematics evolved from it and algebra, and consequently, it forms the basis of modern science, especially physics. It is a powerful tool for solving complex problems in mathematics, physics, economics, the social sciences, engineering, and the biological sciences, among other fields. The importance of calculus is obvious. Therefore, we have written this textbook in the hope that all calculus students will gain a true understanding and mastery of the subject and that it will provide strong support for further development in related fields in the future.

This textbook comprises 16 chapters and is divided into two volumes. To make this textbook fully adaptable, it contains essential contents applicable to different university and college programs, as well as contents marked with \* that can be selected by various universities and colleges.

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