

High School Calculus: A New Perspective

**For Any High School Curriculum,
Including IB, A-Level, AP, and More**

Spring Seeds

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Preface

The first chapter of this book gives the preparatory knowledge on which calculus depends. The second chapter explains the concept of limit and its definition from a dynamic point of view. With this basic knowledge, further study of calculus becomes concrete and straightforward.

We have written this calculus textbook to use in our teaching, and it has received high praise from students. Students found the concepts and theories in this book to be very clear and logical, and thus, learning calculus became very easy, which is easier than learning other mathematics courses. Students also achieved excellent grades in the study of calculus. The teaching effect is remarkable. It can be said that this book has completely changed the difficult situation of teaching calculus and made learning calculus easy.

Calculus is one of the outstanding achievements of the human intellect. It is a powerful tool for solving complex problems in mathematics, physics, economics, the social sciences, engineering, and the biological sciences, among other fields. The importance of calculus is obvious. Therefore, we have written this textbook in the hope that all calculus students will gain a true understanding and mastery of the subject and that it will provide strong support for further development in related fields in the future.

To make this textbook fully adaptable, it contains essential contents applicable to different high school curricula, as well as contents marked with * that can be selected by various high schools.

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